

# Lie symmetries applied to guaranteed integration: application to mobile robotics localisation

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## Introduction

When dealing with mobile robots, localisation is one of the main problems one will be required to solve. One may use complex and expensive devices to localise the vehicle. However, with the development of swarms of robots, the need for cheaper units requires finding alternatives to these expensive devices. Here we will use the dynamical model of the system studied.

## Guaranteed integration using Lie symmetries

In [1] and [2], we presented a method to perform guaranteed integration for large initial conditions. We recall the principle quickly here. Provided we have a reference trajectory ( $\phi_{\mathbf{x}_0^1}(t)$ , black arrow in Fig. 1) computed for an initial condition  $\mathbf{x}_0^1$  (the bottom left black car), it is possible, using Lie symmetries of the system (if they are known) to find the solution (top right green car) for another initial condition (bottom right green car).

## Combining interval analysis tools to solve the localisation problem

Consider a robot with a known dynamical model and an unknown initial state. We want to localise it, using range-only measurements to beacons for which we know the exact location as it progresses throughout the mission. Applying both our integration method and contractors related to range measurements on tubes [3] used to enclose the robot trajectory, it is possible to estimate the initial condition of the robot with backward constraint propagation. Then, propagating the constraints in a forward manner, one can

estimate the trajectory of the robot. This is illustrated in a video which can be found on youtube<sup>1</sup>.

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## References

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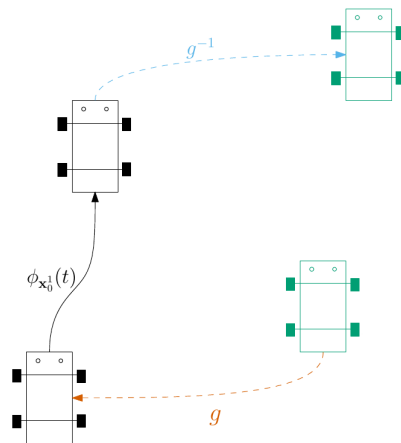


Figure 1: Guaranteed integration principle using Lie symmetries

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<sup>1</sup>State estimation using Lie symmetries by Julien DAMERS